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Multi-Dimensional High Order Essentially Non-Oscillatory
Finite Difference Methods in Generalized Coordinates

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We have investigated the nonlinear stability of compact schemes for shock calculations. In recent years compact schemes have been used in various numerical simulations including direct numerical simulation of turbulence. However to apply them to problems containing shocks, one has to resolve the problem of spurious numerical oscillation and nonlinear instability. We have introduced a framework to apply nonlinear limiting to a local mean. The resulting scheme can be proven total variation (1D) or maximum norm (multi D) stable and produces nice numerical results in the test cases. The result is summarized in the preprint entitled "Nonlinearly Stable Compact Schemes for Shock Calculations", co-authored with Bernardo Cockburn, which has been submitted to SIAM Journal on Numerical Analysis. This paper quoted partial support by NASA Langley grant NAG-1-1145.

Research has been continuing on issues related to two and three dimensional ENO schemes. The main research topics include: (1) parallel implementation of ENO schemes on Connection Machines; (2) boundary conditions; (3) shock interaction with hydrogen bubbles, a preparation for the full combustion simulation; (4) direct numerical simulation of compressible sheared turbulence.

For spectral method, we have obtained results for uniformly spectral accurate reconstruction using Gegenbauer polynomials from the Fourier partial sum of a discontinuous function. This completely overcomes the Gibbs phenomenon. The result is contained in the preprint "On the Gibbs Phenomenon

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I: Recovering Exponential Accuracy from the Fourier Partial Sum of a Non-Periodic Analytic Function", co-authored with D. Gottlieb, A. Solomonoff and H. Vandeven, which will appear in Journal of Applied and Computational Mathematics. This paper also quoted partial support by NASA Langley grant NAG-1-1145.

NONLINEARLY STABLE COMPACT SCHEMES FOR SHOCK CALCULATIONS

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Key Words: compact schemes, total variation stability, maximum principle, shock calculations.

AMS(MOS) subject classification: 65M10

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ABSTRACT

In this paper we discuss the applications of high order compact finite difference methods for shock calculations. The main idea is the definition of a local mean which serves as a reference for introducing a local nonlinear limiting to control spurious numerical oscillations while keeping the formal accuracy of the scheme. For scalar conservation laws, the resulting schemes can be proven total variation stable in one space dimension and maximum norm stable in multi space dimensions. Numerical examples are shown to verify accuracy and stability of such schemes for problems containing shocks. The idea in this paper can also be applied to other implicit schemes such as the continuous Galerkin finite element methods.

ON THE GIBBS PHENOMENON I:
RECOVERING EXPONENTIAL ACCURACY FROM THE FOURIER PARTIAL SUM OF A NON-PERIODIC ANALYTIC FUNCTION [1]

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Key Words: Gibbs phenomenon, Fourier series, Gegenbauer polynomials, exponential accuracy

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ABSTRACT

It is well known that the Fourier series of an analytic and periodic function, truncated after $2N+1$ terms, converges *exponentially* with N, even in the maximum norm. It is also known that if the function is *not* periodic, the rate of convergence deteriorates; in particular there is no convergence in the maximum norm, although the function is still analytic. This is known as the **Gibbs phenomenon**. In this paper we show that the first $2N+1$ Fourier coefficients contain enough information about the function, so that an exponentially convergent approximation (in the *maximum* norm) can be constructed. The proof is a constructive one and makes use of the Gegenbauer polynomials $C_n^\lambda(x)$. It consists of two steps:

In the first step we show that the first m coefficients of the Gegenbauer expansion (based on $C_n^\lambda(x)$, for $0 \le n \le m$) of *any* L_2 function can be obtained, within exponential accuracy, provided that both λ and m are proportional to (but smaller than) N.

In the second step we construct the Gegenbauer expansion based on C_n^λ , $0 \le n \le m$ from the coefficients found in the first step. We show that this series converges exponentially with N, provided that the original function is analytic (though non-periodic).

Thus we prove that : The Gibbs phenomenon can be completely overcome.